

Matrices and Determinants

Miscellaneous Exercise Question Bank

Level - 1

1. Evaluate the following determinants.

(A)
$$\begin{vmatrix} a-b & b-c & c-a \\ p-q & q-r & r-p \\ l-m & m-n & n-l \end{vmatrix}$$

(B)
$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ b+c & c+a & a+b \end{vmatrix}$$

(C)
$$\begin{vmatrix} 1 & ab & c(a+b) \\ 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \end{vmatrix}$$

(D)
$$\begin{vmatrix} c(a-b) & a(b-c) & b(c-a) \\ r(p-q) & p(q-r) & q(r-p) \\ n(l-m) & l(m-n) & m(n-l) \end{vmatrix}$$

(E)
$$\begin{vmatrix} 1 & bc & bc(b+c) \\ 1 & ca & ca(c+a) \\ 1 & ab & ab(a+b) \end{vmatrix}$$

(F)
$$\begin{vmatrix} 1 & ab & \frac{1}{a} + \frac{1}{b} \\ 1 & bc & \frac{1}{b} + \frac{1}{c} \\ 1 & ca & \frac{1}{c} + \frac{1}{a} \end{vmatrix}$$

(G)
$$\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{vmatrix}$$

(H)
$$\begin{vmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{vmatrix}$$

2. Prove the following :

(A)
$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)$$

(B)
$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

(C)
$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$$

(D)
$$\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = (ab+bc+ca)(a-b)(b-c)(c-a)$$

(E)
$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^4 & b^4 & c^4 \end{vmatrix} = (a^2+b^2+c^2+ab+bc+ca)(a-b)(b-c)(c-a)$$

3. Prove the following :

$$(A) \begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix} = 4xyz$$

$$(B) \begin{vmatrix} b^2+c^2 & c^2 & b^2 \\ c^2 & c^2+a^2 & a^2 \\ b^2 & a^2 & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$$

$$(C) \begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$$

$$(D) \begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & cb+b^2 & c^2 \end{vmatrix} = 4a^2b^2c^2$$

4. If a matrix A is such that $4A^3 + 2A^2 + 7A + I = O$, then A^{-1} equals :

$$(A) (4A^2 + 2A + 7I)$$

$$(B) -(4A^2 + 2A + 7I)$$

$$(C) -(4A^2 - 2A + 7I)$$

$$(D) (4A^2 + 2A - 7I)$$

5. For how many value (s) of x in the closed interval $[-4, -1]$ is the matrix $\begin{bmatrix} 3 & -1+x & 2 \\ 3 & -1 & x+2 \\ x+3 & -1 & 2 \end{bmatrix}$ singular :

$$(A) 2$$

$$(B) 0$$

$$(C) 3$$

$$(D) 1$$

6. If $3, -2$ are the Eigen values of a non-singular matrix A and $|A| = 4$, then the Eigen values of $\text{adj}(A)$ are :

$$(A) \frac{3}{4}, \frac{-1}{2}$$

$$(B) \frac{4}{3}, -2$$

$$(C) 12, -8$$

$$(D) -12, 8$$

7. Let p a non singular matrix $1 + P + P^2 + \dots + P^n = O$. (O denotes the null matrix), then $P^{-1} =$

$$(A) P^n$$

$$(B) -P^n$$

$$(C) -(1 + P + \dots + P^n)$$

$$(D) \text{None of these}$$

8. If $P = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$ $\begin{bmatrix} -1 & -2 \\ -2 & 0 \\ 0 & -4 \end{bmatrix}$ $\begin{bmatrix} -4 & -5 & -6 \\ 0 & 0 & 1 \end{bmatrix}$ then $P_{22} =$

$$(A) 40$$

$$(B) -40$$

$$(C) -20$$

$$(D) 20$$

9. If $1, \omega, \omega^2$ are the cube roots of unity, then $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ is equal to :

$$(A) 0$$

$$(B) 1$$

$$(C) \omega$$

$$(D) \omega^2$$

10. If $A = \begin{vmatrix} \sin(\theta + \alpha) & \cos(\theta + \alpha) & 1 \\ \sin(\theta + \beta) & \cos(\theta + \beta) & 1 \\ \sin(\theta + \gamma) & \cos(\theta + \gamma) & 1 \end{vmatrix}$, then :

$$(A) A = 0 \text{ for all } \theta$$

$$(B) A \text{ is an odd Function of } \theta$$

$$(C) A = 0 \text{ for } \theta = \alpha + \beta + \gamma$$

$$(D) A \text{ is independent of } \theta$$

11. Let $\Delta_1 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{vmatrix}$, then $\Delta_1 \times \Delta_2$ can be expressed as the sum of how many determinants :
- (A) 9 (B) 3 (C) 27 (D) 2
12. If $a + b + c = 0$, then the solution of the equation $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$ is :
- (A) 0 (B) $\pm \frac{3}{2}(a^2 + b^2 + c^2)$
 (C) $0, \pm \sqrt{\frac{3}{2}(a^2 + b^2 + c^2)}$ (D) $0, \pm \sqrt{a^2 + b^2 + c^2}$
13. The value of a for which the system of equations $a^3x + (a+1)^3y + (a+2)^3z = 0$, $ax + (a+1)y + (a+2)z = 0$, $x + y + z = 0$, has a non zero solution is :
- (A) -1 (B) 0 (C) 1 (D) None of these
14. If $a_1x + b_1y + c_1z = 0$, $a_2x + b_2y + c_2z = 0$, $a_3x + b_3y + c_3z = 0$ and $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$, then the given system has :
- (A) One trivial and one non-trivial solution (B) No solution
 (C) One solution (D) Infinite solution
15. If $\begin{bmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ x & 1 & -5 \end{bmatrix}$ is a singular matrix, then x is :
- (A) $\frac{13}{25}$ (B) $-\frac{25}{13}$ (C) $\frac{5}{13}$ (D) $\frac{25}{13}$
16. For each real number x such that $-1 < x < 1$, let $A(x)$ be the matrix $(1-x)^{-1} \begin{bmatrix} 1 & -x \\ -x & 1 \end{bmatrix}$ and $z = \frac{x+y}{1+xy}$. Then :
- (A) $A(z) = A(x) + A(y)$ (B) $A(z) = A(x)[A(y)]^{-1}$
 (C) $A(z) = A(x)A(y)$ (D) $A(z) = A(x) - A(y)$
17. If matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & 6 & 7 \end{bmatrix}$ and its inverse is denoted by $A^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, then the value of $a_{23} =$
- (A) $\frac{21}{20}$ (B) $\frac{1}{5}$ (C) $-\frac{2}{5}$ (D) $\frac{2}{5}$

18. Let $F(\alpha) = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, where $\alpha \in R$. Then $[F(\alpha)]^{-1}$ is equal to :
- (A) $F(-\alpha)$ (B) $F(\alpha^{-1})$ (C) $F(2\alpha)$ (D) None of these
- *19. If A is unimodular, then which of the following is unimodular?
- (A) $-A$ (B) A^{-1}
(C) $\text{adj } A$ (D) ωA , where ω is cube root of unity
- *20. If $A = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A+B)^2 = A^2 + B^2 + 2AB$, then :
- (A) $a = -1$ (B) $a = 1$ (C) $b = 2$ (D) $b = -2$
- *21. If $AB = A$ and $BA = B$, then which of the following is/are true?
- (A) A is idempotent (B) B is idempotent
(C) A^T is idempotent (D) None of these
- *22. If $A = \frac{1}{3} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is an orthogonal matrix, then :
- (A) $a = -2$ (B) $a = 2, b = 1$ (C) $b = -1$ (D) $b = 1$
- *23. If A is a matrix such that $A^2 + A + 2I = O$, then which of the following is/are true?
- (A) A is nonsingular (B) A is symmetric
(C) A cannot be skew-symmetric (D) $A^{-1} = -\frac{1}{2}(A + I)$
- *24. If $\begin{pmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{pmatrix} \begin{pmatrix} 1 & \tan \theta \\ -\tan \theta & 1 \end{pmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then :
- (A) $a = \cos 2\theta$ (B) $a = 1$ (C) $b = \sin 2\theta$ (D) $b = -1$
- *25. If B is an idempotent matrix, and $A = I - B$, then ;
- (A) $A^2 = A$ (B) $A^2 = I$ (C) $AB = O$ (D) $BA = O$
- *26. If $A_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$, $A_2 = \begin{bmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{bmatrix}$. Then $A_i A_k + A_k A_i$ is equal to :
- (A) $2I$ if $i = k$ (B) O if $i \neq k$ (C) $2I$ if $i \neq k$ (D) 0 always
- *27. If $A = (a_{ij})_{n \times n}$ and f is a function, we define $f(A) = (f(a_{ij}))_{n \times n}$.
- Let $A = \begin{pmatrix} \pi/2 - \theta & \theta \\ -\theta & \pi/2 - \theta \end{pmatrix}$. Then :
- (A) $\sin A$ is invertible (B) $\sin A = \cos A$
(C) $\sin A$ is orthogonal (D) $\sin(2A) = 2 \sin A \cos A$

- *28.** If A an invertible matrix, then $(\text{adj. } A)^{-1}$ is equal to :
- (A) $\text{adj.}(A^{-1})$ (B) $\frac{A}{\det. A}$ (C) A (D) $(\det. A)A$
- *29.** If A and B are two invertible matrices of the same order, then $\text{adj}(AB)$ is equal to :
- (A) $\text{adj}(B) \text{adj}(A)$ (B) $|B| |A| B^{-1}A^{-1}$ (C) $|B| |A| A^{-1}B^{-1}$ (D) $|A| |B| (AB)^{-1}$
- *30.** If α, β, γ are three real numbers and $A = \begin{bmatrix} 1 & \cos(\alpha - \beta) & \cos(\alpha - \gamma) \\ \cos(\beta - \alpha) & 1 & \cos(\beta - \gamma) \\ \cos(\gamma - \alpha) & \cos(\gamma - \beta) & 1 \end{bmatrix}$, then which of following is/are true ?
- (A) A is singular (B) A is symmetric (C) A is orthogonal (D) A is not invertible
- *31.** If D_1 and D_2 are two 3×3 diagonal matrices, then which of the following is/are true ?
- (A) $D_1 D_2$ is a diagonal matrix (B) $D_1 D_2 = D_2 D_1$
- (C) $D_1^2 + D_2^2$ is a diagonal matrix (D) None of these
- *32.** If A and B are symmetric and commute, then which of the following is/are symmetric ?
- (A) $A^{-1}B$ (B) AB^{-1} (C) $A^{-1}B^{-1}$ (D) None of these
- *33.** A skew-symmetric matrix A satisfies the relation $A^2 + I = O$, where I is a unit matrix then A is :
- (A) idempotent (B) orthogonal (C) of even order (D) odd order
- 34.** Prove that $\begin{vmatrix} a_1\alpha_1 + b_1\beta_1 & a_1\alpha_2 + b_1\beta_2 & a_1\alpha_3 + b_1\beta_3 \\ a_2\alpha_1 + b_2\beta_1 & a_2\alpha_2 + b_2\beta_2 & a_2\alpha_3 + b_2\beta_3 \\ a_3\alpha_1 + b_3\beta_1 & a_3\alpha_2 + b_3\beta_2 & a_3\alpha_3 + b_3\beta_3 \end{vmatrix} = 0$
- 35.** If $\begin{vmatrix} 1 & x & x^2 \\ x & x^2 & 1 \\ x^2 & 1 & x \end{vmatrix} = 3$, then find the value of $\begin{vmatrix} x^3 - 1 & 0 & x - x^4 \\ 0 & x - x^4 & x^3 - 1 \\ x - x^4 & x^3 - 1 & 0 \end{vmatrix}$
- *36.** A is a matrix of order 3×3 formed using five times 2 and four times 1, then :
- (A) Total number of different matrices is 126
- (B) Total number of symmetric matrices A is 12
- (C) If A is a symmetric matrix, probability that A is singular is $2/3$
- (D) Number of matrices A with maximum trace is 15

Miscellaneous Exercise Question Bank

Level - 2

37. Let $A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & -2 & 5 \\ 0 & 4 & 3 \end{pmatrix}$, $B = \text{Adj}(\text{Adj}(A)) = (b_{ij})_{3 \times 3}$ then match the following

Column 1		Column 2	
(A)	Trace (B)	(p)	1
(B)	$\sum_{i=1}^3 \sum_{j=1}^3 b_{ij}$	(q)	-40
(C)	$\frac{ \text{Adj}(A) \cdot B }{ A ^2}$	(r)	-240
(D)	$\frac{ B }{ \text{Adj} A }$	(s)	400

38. Match the Column:

Column 1		Column 2	
(A)	$2x + y - 3z = 1$, $x - 2y + z = -4$, $x + 8y - 9z = 14$	(p)	Simultaneous equations represents a point
(B)	$x + 5y + 2z = -1$, $3x - y + z = 2$, $x + 21y + 7z = 6$	(q)	Simultaneous equations represents a line
(C)	$2x - 3y + z = 0$, $3x + y - 2z = 0$, $4x + 2y - 5z = 0$	(r)	Simultaneous equations represents a plane
(D)	$2ax + by + 4cz = 0$, $bx + 4cy + 2az = 0$, $4cx + 2ay + bz = 0$ $4a^2 + b^2 + 16c^2 - 2ab - 4bc - 8ac = 0$, $a, b, c \in \mathbb{R}$ $a \neq 0$	(s)	Simultaneous equations represents a triangular prism

39. Let S be set of matrices defined as $S = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$: $abcd$ denotes all possible 4 digit decimal number) also

define for some $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix} \in S$, $D(A) = ps - qr$, $T(A) = p + s$, $T'(A) = q + r$

Match the column :

Column 1		Column 2	
(A)	Number of singular matrices in S which have no zero entry × Number of singular matrices in S which have atleast one zero entry	(p)	20250
(B)	$\sum_{A \in S} D(A)$	(q)	85500
(C)	$\sum_{A \in S} T(A)$	(r)	35739
(D)	$\sum_{A \in S} T'(A)$	(s)	81000

40. Let $A_k = \begin{bmatrix} {}^nC_{k-1} & 0 \\ 0 & {}^nC_k \end{bmatrix}$ & $B = \sum_{k=1}^{n-1} A_k \times A_{k+1} + nI_2 = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

Column 1		Column 2	
(A)	a	(p)	$\frac{2n}{2n+1} \binom{2n+1}{n} C_n$
(B)	$a - b$	(q)	0
(C)	$a + b$	(r)	${}^{2n}C_{n+1}$
(D)	a/b	(s)	1

41. A and B are 2 non singular matrices such that $(AB)^k = A^k B^k$ for 3 consecutive positive integral values of k .



Column 1		Column 2	
(A)	ABA^{-1}	(p)	A^2
(B)	BAB^{-1}	(q)	B
(C)	$ABAB^{-1}$	(r)	A
(D)	$B(BA)^{-1}AB^2$	(s)	B^2

42.

Column I			Column II	
(A)	The value of the determinant	$\begin{vmatrix} x+2 & x+3 & x+5 \\ x+4 & x+6 & x+9 \\ x+8 & x+11 & x+15 \end{vmatrix}$ is	(p)	1
(B)	If one of the roots of the equation	$\begin{vmatrix} 7 & 6 & x^2-13 \\ 2 & x^2-13 & 2 \\ x^2-13 & 3 & 7 \end{vmatrix} = 0$ is $x+2$, then sum of all other five roots is	(q)	-6
(C)	The value of	$\begin{vmatrix} \sqrt{6} & 2i & 3+\sqrt{6} \\ \sqrt{12} & \sqrt{3}+\sqrt{8}i & 3\sqrt{2}+\sqrt{6}i \\ \sqrt{18} & \sqrt{2}+\sqrt{12}i & \sqrt{27}+2i \end{vmatrix}$ is	(r)	2
(D)	If $f(\theta) = \begin{vmatrix} \cos^2 \theta & \cos \theta \sin \theta & -\sin \theta \\ \cos \theta \sin \theta & \sin^2 \theta & \cos \theta \\ \sin \theta & -\cos \theta & 0 \end{vmatrix}$, then $f(\pi/3)$		(s)	-2

43.

Column I			Column II	
(A)	$\begin{vmatrix} 1/c & 1/c & -(a+b)/c^2 \\ -(b+c)/a^2 & 1/a & 1/a \\ -b(b+c)/a^2c & (a+2b+c)/ac & -b(a+b)/ac^2 \end{vmatrix}$	is	(p)	Independent of a
(B)	$\begin{vmatrix} \sin a \cos b & \sin a \sin b & \cos a \\ \cos a \cos b & \cos a \sin b & -\sin a \\ -\sin a \sin b & \sin a \cos b & 0 \end{vmatrix}$	is	(q)	Independent of b
(C)	$\begin{vmatrix} \frac{1}{\sin a \cos b} & \frac{1}{\sin a \sin b} & \frac{1}{\cos a} \\ -\cos a & -\cos a & \sin a \\ \frac{\sin^2 a \cos b}{\sin b} & \frac{\sin^2 a \sin b}{-\cos b} & \cos^2 a \\ \sin a \cos^2 b & \sin a \sin^2 b & 0 \end{vmatrix}$	is	(r)	Independent of c
(D)	If a, b, and c are the sides of a triangle and A, B and C are the angles opposite to a, b, and c, respectively, then $\Delta = \begin{vmatrix} a^2 & b \sin A & c \sin A \\ b \sin A & 1 & \cos A \\ c \sin A & \cos A & 1 \end{vmatrix}$		(s)	Dependent on a, b

44.



Column I		Column II	
(A)	$(I - A)^n$ is if A is idempotent	(p)	$2^{n-1}(I - A)$
(B)	$(I - A)^n$ is if A is involutory	(q)	$I - nA$
(C)	$(I - A)^n$ is if A is nilpotent of index 2	(r)	A
(D)	If A is orthogonal, then $(A^T)^{-1}$	(s)	$I - A$

45.

Column I		Column II	
(A)	If A is an idempotent matrix and I is an identity matrix of the same order, then the value of n, such that $(A + I)^n = I + 127$ is	(p)	9
(B)	If $(I - A)^{-1} = I + A + A^2 + \dots + A^7$, then $A^n = 0$, where n is	(q)	10
(C)	If A matrix such that $a_{ij} = (i + j)(i - j)$, then A is singular if order of matrix is	(r)	7
(D)	If a nonsingular matrix A is symmetric, shown that A^{-1} is also symmetric, then order of A can be	(s)	8

46.

Column I		Column II	
(A)	If $ A = 2$, then $ 2A^{-1} =$ (where A is of order 3)	(p)	1
(B)	If $ A = 1/8$, then $ adj(adj(2A)) =$ (where A is of order 3)	(q)	4
(C)	If $(A+B)^2 = A^2 + B^2$, and $ A = 2$, then $ B =$ (where A and B are of odd order)	(r)	24
(D)	$ A_{2 \times 2} = 2, B_{3 \times 3} = 3$ and $ C_{4 \times 4} = 4$, then $ ABC $ is equal to	(s)	0
		(t)	Does not exist

Paragraph for Questions 47 – 49

Let A be a matrix of order 2×2 such that $A^2 = 0$.

47. $A^2 - (a+d)A + (ad-bc)I$ is equal to:
 (A) I (B) O (C) $-I$ (D) None of these
48. $tr(A)$ is equal to:
 (A) 1 (B) 0 (C) -1 (D) None of these
49. $(I+A)^{100} =$
 (A) $100A$ (B) $100(I+A)$ (C) $100I + A$ (D) $I + 100A$

Paragraph for Questions 50 – 52

If A and B are two square matrices of order 3×3 which satisfy $AB = A$ and $BA = B$, then:

50. Which of the following is true?
 (A) If matrix A is singular, then matrix B is nonsingular.
 (B) If matrix A is nonsingular, then matrix B is singular.
 (C) If matrix A is singular, then matrix B is also singular.
 (D) Cannot say anything.
51. $(A+B)^7$ is equal to:
 (A) $7(A+B)$ (B) $7 \cdot I_{3 \times 3}$ (C) $64(A+B)$ (D) $128I$
52. $(A+I)^5$ is equal to (where I is identity matrix)
 (A) $I + 60A$ (B) $I + 16A$ (C) $I + 31A$ (D) None of these

Paragraph for Questions 53 – 54

Consider an arbitrary 3×3 matrix $A = [a_{ij}]$. A matrix $B = [b_{ij}]$ is formed such that b_{ij} is the sum of all the elements except a_{ij} in the i th row of A . Answer the following questions:

53. If there exists a matrix X with constant elements such that $AX = B$, then X is:
 (A) skew-symmetric (B) null matrix
 (C) diagonal matrix (D) none of these
54. The value of $|B|$ is equal to:
 (A) $|A|$ (B) $|A|/2$ (C) $2|A|$ (D) none of these



Paragraph for Questions 55 – 57

Let $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ satisfies $A^n = A^{n-2} + A^2 - I$ for $n \geq 3$. And trace of a square matrix X is equal to the sum of elements in its principal diagonal.

Further consider a matrix U with its column as U_1, U_2, U_3 such that

$$A^{50}U_1 = \begin{bmatrix} 1 \\ 25 \\ 25 \end{bmatrix}, A^{50}U_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, A^{50}U_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Then answer the following questions:

- 55.** The value of $|A^{50}|$ equals:
(A) 0 **(B)** 1 **(C)** -1 **(D)** 25
- 56.** Trace of A^{50} equals:
(A) 0 **(B)** 1 **(C)** 2 **(D)** 3
- 57.** The value of $|U|$ equals:
(A) 0 **(B)** 1 **(C)** 2 **(D)** -1

Paragraph for Questions 58 – 60

Let A be a square matrix of order 2 or 3 and I be the identity matrix of the same order. Then the matrix $A - \lambda I$ is called characteristic matrix of the matrix A , where λ is some complex number. The determinant of the characteristic matrix is called characteristic determinant of the matrix A which will of course be a polynomial of degree 3 in λ . The equation $\det(A - \lambda I) = 0$ is called characteristic equation of the matrix A and its roots (the value of λ) are called characteristic roots or eigen values. It is also known that every square matrix has its characteristic equation.

- 58.** The eigenvalues of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{bmatrix}$ are:
(A) 2, 1, 1 **(B)** 2, 3, -2 **(C)** -1, 1, 3 **(D)** None of these
- 59.** Which of the following matrices do not have eigenvalues as 1 and -1 ?
(A) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ **(B)** $\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ (where $i = \sqrt{-1}$)
(C) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ **(D)** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- 60.** If one of the eigenvalues of a square matrix A of order 3×3 is zero, then:
(A) $\det A$ must be nonzero **(B)** $\det A$ must be zero
(C) $\text{adj } A$ must be a zero matrix **(D)** None of these

Paragraph for Questions 61 – 63

Let A be an $m \times n$ matrix. If there exists a matrix L of type $n \times m$ such that $LA = I_n$, then L is called left inverse of A . Similarly, if there exists a matrix R of type $n \times m$ such that $AR = I_m$, then R is called right inverse of A . For example, to find right inverse of matrix.

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix}, \text{ we take } R = \begin{bmatrix} x & y & z \\ u & v & w \end{bmatrix} \text{ and solve } AR = I_3, \text{ i.e., } \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x & y & z \\ u & v & w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{array}{lll} x - u = 1 & y - v = 0 & z - w = 0 \\ x + u = 0 & y + v = 1 & z + w = 0 \\ 2x + 3u = 0 & 2y + 3v = 0 & 2z + 3w = 1 \end{array}$$

As this system of equations is inconsistent, we say there is no right inverse for matrix A .

61. Which of the following matrices is NOT left inverse of matrix $\begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix}$?

(A) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & -7 & 3 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ (C) $\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 3 & -1 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$

62. The number of right inverses for the matrix $\begin{bmatrix} 1 & -1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$ is:

(A) 0 (B) 1 (C) 2 (D) infinite

63. For which of the following matrices, the number of left inverses is greater than the number of right inverses?

(A) $\begin{bmatrix} 1 & 2 & 4 \\ -3 & 2 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 3 & 2 & 1 \\ 3 & 2 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 4 \\ 2 & -3 \\ 5 & 4 \end{bmatrix}$ (D) $\begin{bmatrix} 3 & 3 \\ 1 & 1 \\ 4 & 4 \end{bmatrix}$

Paragraph for Questions 64 - 66



Suppose a, b, c are 3 distinct real numbers and $f(x)$ be a quadratic function which satisfies

$$\begin{bmatrix} a^2 & a & 1 \\ b^2 & b & 1 \\ c^2 & c & 1 \end{bmatrix} \begin{bmatrix} f(-1) \\ f(2) \\ f(4) \end{bmatrix} = \begin{bmatrix} a - 8a^2 - 3 \\ b - 8b^2 - 3 \\ c - 8c^2 - 3 \end{bmatrix}$$

The maximum of $f(x)$ occurs at V . A is a point of $y = f(x)$ where it cuts y axis, B is a point on $y = f(x)$ such

that $\angle AVB$ is 90° , $A(x_1, y_1), V(x_2, y_2), B(x_3, y_3)$ and $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, Y = (y_1 y_2 y_3)$ then:

64. $\begin{vmatrix} f(-1) & f(0) & f(1) \\ f(0) & f(1) & f(2) \\ f(1) & f(2) & f(3) \end{vmatrix} =$

- (A) $4 \times \text{slope of } AV$ (B) $-4 \times \text{slope of } AV$
 (C) 0 (D) $1/2 \text{ slope of } AV$

65. Trace of XY is :

- (A) $31/8$ (B) $23/4$ (C) $17/8$ (D) $7/8$

66. $XY \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$ has :

- (A) unique solution (B) no solution
(C) Infinitely many solutions lying on a line (D) infinitely many solutions lying on a plane

Paragraph for Questions 67 - 69

Consider a set A of matrices: $\left\{ \begin{bmatrix} a & c \\ c & b \end{bmatrix} : a, b, c \in \{0, 1, 2, 3, 4, 5, 6\} \right\}$

67. Number of matrices in A which are symmetric, or skew symmetric but not both is :

- (A) 343 (B) 342 (C) 49 (D) 336

68. Number of matrices in A whose determinant is divisible by 7 is :

- (A) 49 (B) 36 (C) 294 (D) 330

69. Number of matrices in A such that its trace and its determinant both divisible by 7 is : (trace is sum of diagonal elements)

- (A) 0 (B) 1 (C) 3 (D) 48

Paragraph for Questions 70 - 72



Consider function $f(\lambda) = |A_n - \lambda I_n|$ where A_n is square matrix of order n , I_n is identity matrix of order n .

If $\lambda \in \mathbb{C}$ (set of complex numbers) then roots of $f(\lambda) = 0$ are called eigen values of matrix A_n . Also $f(A_n) = 0$.

If $\lambda_1 \in \mathbb{C}$, be such that $f(\lambda_1) = 0$, then column matrix $X_{n \times 1}$ such that $AX = \lambda_1 X$ is known as eigen vector

corresponding to eigen value λ_1 , $A = \begin{pmatrix} 1 & -2 & 3 & 0 \\ -3 & 2 & 1 & \\ 3 & -1 & 0 & \end{pmatrix}$.

70. Eigen value of matrix A are :

- (A) $1, -1 \pm \sqrt{2}$ (B) $1, 1 \pm \sqrt{2}$ (C) $-1, 1 \pm \sqrt{2}$ (D) $1, 1 \pm \sqrt{2}i$

71. Matrix A satisfies the equation :

- (A) $A^{-1} = A^2 + 3A - I$ (B) $A^{-1} = -\frac{1}{3}A^3 + \frac{8}{3}A - \frac{4}{3}I$
(C) $A^{-1} = -A^2 + 3A + I$ (D) $A^{-1} = -\frac{1}{3}A^3 - \frac{10}{3}A + \frac{2}{3}I$

72. Eigen vector corresponding to some eigen value of A is :

- (A) $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ (B) $\begin{pmatrix} 1 \\ \sqrt{2} + 1 \\ 3 \end{pmatrix}$ (C) $\begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix}$ (D) $\begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$

Paragraph for Questions 73 - 75

Let S be the set of all 3×3 symmetric matrices all of whose entries are either 0 or -1 . Five of them are 0 and four are -1 .

73. Number of matrices in S which are non singular is :
 (A) less than 4 (B) at least 4 but less than 7
 (C) at least 7 but less than 10 (D) atleast 10
74. Number of matrices A in S such that $A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ have infinitely many solutions, is :
 (A) 0 (B) More than 2 (C) 2 (D) 1
75. Number of matrices A in S such that $A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ have no solution, is :
 (A) 0 (B) more than 2 (C) 2 (D) 1

Paragraph for Questions 76 - 78

$Adj(A) = (C_{ij})_{n \times n}$, C_{ij} is cofactor of a_{ij} & $A. Adj(A) = |A| I_n$

76. If $\begin{vmatrix} 1 & x & x^2 \\ x & x^2 & 1 \\ x^2 & 1 & x \end{vmatrix} = 7$, then $\begin{vmatrix} x^3 - 1 & 0 & x - x^4 \\ 0 & x - x^4 & x^3 - 1 \\ x - x^4 & x^3 - 1 & 0 \end{vmatrix} =$
 (A) 7 (B) 343 (C) -49 (D) 49
77. Suppose $a, b, c \in R$, $a + b + c > 0$, $A = bc - a^2$, $B = ca - b^2$, $C = ab - c^2$,
 $\begin{vmatrix} A & B & C \\ B & C & A \\ C & A & B \end{vmatrix} = 49$, then $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} =$
 (A) -7 (B) 2401 (C) -2401 (D) 7
78. If A is square matrix of order 3 then $Adj(Adj(Adj(A))) =$
 (A) $|A|^2 A$ (B) $|A|^2 Adj(A)$ (C) $|A|^4 Adj(A)$ (D) $|A|^8 Adj A$

Paragraph for Questions 79 - 81

$f(x) = \begin{vmatrix} x+c_1 & x+a & x+a \\ x+b & x+c_2 & x+a \\ x+b & x+b & x+c_3 \end{vmatrix}$ and $g(x) = (c_1 - x)(c_2 - x)(c_3 - x)$

79. Coefficient of x in $f(x)$ is :
 (A) $\frac{g(a) - f(b)}{b - a}$ (B) $\frac{g(-a) - g(-b)}{b - a}$ (C) $\frac{g(a) - g(b)}{b - a}$ (D) None of these
80. Which of the following is not a constant term in $f(x)$?
 (A) $\frac{bg(a) - ag(b)}{(b - a)}$ (B) $\frac{bg(a) - af(-b)}{(b - a)}$ (C) $\frac{bf(-a) - ag(b)}{(b - a)}$ (D) None of these
81. Which of the following is not true ?
 (A) $f(-a) = g(a)$ (B) $f(-a) = g(-a)$ (C) $f(-b) = g(b)$ (D) None of these

Paragraph for Questions 82 - 84

Given that the system of equations $x = cy + bz$, $y = az + cx$, $z = bx + ay$ has non-zero solutions and at least one of the a, b, c is a proper fraction.

82. $a^2 + b^2 + c^2$ is :
 (A) > 2 (B) > 3 (C) < 3 (D) < 2
83. abc is :
 (A) > -1 (B) > 1 (C) < 2 (D) < 3
84. System has solution such that :
 (A) $x : y : z = (1 - 2a^2) : (1 - 2b^2) : (1 - 2c^2)$ (B) $x : y : z = \frac{1}{1 - 2a^2} : \frac{1}{1 - 2b^2} : \frac{1}{1 - 2c^2}$
 (C) $x : y : z = \frac{a}{1 - a^2} : \frac{b}{1 - b^2} : \frac{c}{1 - c^2}$ (D) $x : y : z = \sqrt{1 - a^2} : \sqrt{1 - b^2} : \sqrt{1 - c^2}$

Paragraph for Questions 85 - 87

Consider the system of equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$

85. The system has unique solution if :
 (A) $\lambda \neq 3$ (B) $\lambda = 3, \mu = 10$ (C) $\lambda = 3, \mu \neq 10$ (D) None of these
86. The system has infinite solutions if :
 (A) $\lambda \neq 3$ (B) $\lambda = 3, \mu = 10$ (C) $\lambda = 3, \mu \neq 10$ (D) None of these
87. The system has no solutions if :
 (A) $\lambda \neq 3$ (B) $\lambda = 3, \mu = 10$ (C) $\lambda = 3, \mu \neq 10$ (D) None of these

Paragraph for Questions 88 - 90

Consider the polynomial function $f(x) = \begin{vmatrix} (1+x)^a & (1+2x)^b & 1 \\ 1 & (1+x)^a & (1+2x)^b \\ (1+2x)^b & 1 & (1+x)^a \end{vmatrix}$, a, b being positive integres.

88. The constant term in $f(x)$ is :
 (A) 2 (B) 1 (C) -1 (D) 0
89. The coefficient of x in $f(x)$ is :
 (A) 2^a (B) $2^a - 3 \times 2^b + 1$ (C) 0 (D) None of these
90. Which of the following is true ?
 (A) All roots of the equation $f(x) = 0$ are positive
 (B) All roots of the equation $f(x) = 0$ are negative
 (C) At least one of the equation $f(x) = 0$ is repeating one
 (D) None of these

Paragraph for Questions 91 - 93

If $x > m, y > n, z > r (x, y, z > 0)$ such that $\begin{vmatrix} x & n & r \\ m & y & r \\ m & n & z \end{vmatrix} = 0$.

- 91.** The value of $\frac{x}{x-m} + \frac{y}{y-n} + \frac{z}{z-r}$ is :
(A) 1 **(B)** -1 **(C)** 2 **(D)** -2
- 92.** The value of $\frac{m}{x-m} - 1 + \frac{y}{y-n} - 1 + \frac{r}{z-r}$ is :
(A) -2 **(B)** -4 **(C)** 0 **(D)** -1
- 93.** The greatest value of $\frac{xyz}{(x-m)(y-n)(z-r)}$ is :
(A) 27 **(B)** $\frac{8}{27}$ **(C)** $\frac{64}{27}$ **(D)** None of these

Paragraph for Questions 94 - 96

Suppose $f(x)$ is a function satisfying the following conditions :

(i) $f(0) = 2, f(1) = 1$

(ii) f has a minimum value at $x = 5/2$

(iii) For all $x, f'(x) = \begin{vmatrix} 2ax & 2ax-1 & 2ax+b+1 \\ b & b+1 & -1 \\ 2(ax+b) & 2ax+2b+1 & 2ax+b \end{vmatrix}$

- 94.** The value of $f(2)$ is :
(A) $\frac{1}{4}$ **(B)** $\frac{1}{2}$ **(C)** -1 **(D)** 3
- 95.** $f(x) = 0$ has :
(A) both roots positive **(B)** both roots negative
(C) roots of opposite sign **(D)** imaginary roots
- 96.** Range of $f(x)$ is :
(A) $[7/16, \infty)$ **(B)** $(-\infty, 15/16]$ **(C)** $[3/4, \infty)$ **(D)** None of these
- *97.** Which of the following has/have value equal to zero ?

(A) $\begin{vmatrix} 8 & 2 & 7 \\ 12 & 3 & 5 \\ 16 & 4 & 3 \end{vmatrix}$

(B) $\begin{vmatrix} 1/a & a^2 & bc \\ 1/b & b^2 & ac \\ 1/c & c^2 & ab \end{vmatrix}$

(C) $\begin{vmatrix} a+b & 2a+b & 3a+b \\ 2a+b & 3a+b & 4a+b \\ 4a+b & 5a+b & 6a+b \end{vmatrix}$

(D) $\begin{vmatrix} 2 & 43 & 6 \\ 7 & 35 & 4 \\ 3 & 17 & 2 \end{vmatrix}$

*98. If $g(x) = \begin{vmatrix} a^{-x} & e^{x \log_e a} & x^2 \\ a^{-3x} & e^{3x \log_e a} & x^4 \\ a^{-5x} & e^{5x \log_e a} & 1 \end{vmatrix}$, then :

- (A) graph of $g(x)$ is symmetrical about the origin
(B) graph of $g(x)$ is symmetrical about the y-axis

(C) $\left. \frac{d^4 g(x)}{dx^4} \right|_{x=0} = 0$

(D) $f(x) = g(x) \times \log \left(\frac{a-x}{a+x} \right)$ is an odd function

*99. If $\Delta = \begin{vmatrix} -x & a & b \\ b & -x & a \\ a & b & -x \end{vmatrix}$, then a factor of Δ is :

(A) $a+b+x$

(B) $x^2 - (a-b)x + a^2 + b^2 + ab$

(C) $x^2 + (a+b)x + a^2 + b^2 - ab$

(D) $a+b-x$

*100. If $\Delta = \begin{vmatrix} \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \\ \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\ -\sin \theta \sin \phi & \sin \theta \cos \phi & 0 \end{vmatrix}$ then :

(A) Δ is independent of θ

(B) Δ is independent of ϕ

(C) Δ is a constant

(D) $\left. \frac{d\Delta}{d\theta} \right|_{\theta=\pi/2} = 0$

*101. If $\phi(\alpha, \beta) = \begin{vmatrix} \cos \alpha & -\sin \alpha & 1 \\ \sin \alpha & \cos \alpha & 1 \\ \cos(\alpha+\beta) & -\sin(\alpha+\beta) & 1 \end{vmatrix}$, then :

(A) $f(300, 200) = f(400, 200)$

(B) $f(200, 400) = f(200, 600)$

(C) $f(100, 200) = f(200, 200)$

(D) None of these

*102. The determinant $\begin{vmatrix} a^2+x & ab & ac \\ ab & b^2+x & bc \\ ac & bc & c^2+x \end{vmatrix}$ is divisible by :

(A) x

(B) x^2

(C) x^3

(D) None of these

*103. If $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, then $f(2x) - f(x)$ is divisible by :

(A) x

(B) a

(C) $2a+3x$

(D) x^2

*104. $\Delta = \begin{vmatrix} 1 & 1+ac & 1+bc \\ 1 & 1+ad & 1+bd \\ 1 & 1+ae & 1+be \end{vmatrix}$ is independent of :

(A) a

(B) b

(C) c, d, e

(D) None of these

***105.** If $g(x) = \frac{f(x)}{(x-a)(x-b)(x-c)}$, where $f(x)$ is a polynomial of degree < 3 , then :



(A) $\int g(x)dx = \begin{vmatrix} 1 & a & f(a)\log|x-a| \\ 1 & b & f(b)\log|x-b| \\ 1 & c & f(c)\log|x-c| \end{vmatrix} \div \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + k$

(B) $\frac{dg(x)}{dx} = \begin{vmatrix} 1 & a & f(a)(x-a)^{-2} \\ 1 & b & f(b)(x-b)^{-2} \\ 1 & c & f(c)(x-c)^{-2} \end{vmatrix} \div \begin{vmatrix} a^2 & a & 1 \\ b^2 & b & 1 \\ c^2 & c & 1 \end{vmatrix}$

(C) $\frac{dg(x)}{dx} = \begin{vmatrix} 1 & a & f(a)(x-a)^{-2} \\ 1 & b & f(b)(x-b)^{-2} \\ 1 & c & f(c)(x-c)^{-2} \end{vmatrix} \div \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

(D) $\int g(x)dx = \begin{vmatrix} 1 & a & f(a)\log|x-a| \\ 1 & b & f(b)\log|x-b| \\ 1 & c & f(c)\log|x-c| \end{vmatrix} \div \begin{vmatrix} a^2 & a & 1 \\ b^2 & b & 1 \\ c^2 & c & 1 \end{vmatrix} + k$

***106.** If $\Delta(x) = \begin{vmatrix} x^2 + 4x - 3 & 2x + 4 & 13 \\ 2x^2 + 5x - 9 & 4x + 5 & 26 \\ 8x^2 - 6x + 1 & 16x - 6 & 104 \end{vmatrix} = ax^3 + bx^2 + cx + d$, then :

- (A) $a = 3$ (B) $b = 0$ (C) $c = 0$ (D) None of these

***107.** If $f(\theta) = \begin{vmatrix} \sin \theta & \cos \theta & \sin \theta \\ \cos \theta & \sin \theta & \cos \theta \\ \cos \theta & \sin \theta & \sin \theta \end{vmatrix}$, then :

(A) $f(\theta) = 0$ has exactly 2 real solutions in $[0, \pi]$

(B) $f(\theta) = 0$ has exactly 3 real solutions in $[0, \pi]$

(C) range of function $\frac{f(\theta)}{1 - \sin 2\theta}$ is $[-\sqrt{2}, \sqrt{2}]$

(D) range of function $\frac{f(\theta)}{\sin 2\theta - 1}$ is $[-3, 3]$

***108.** If $f(\theta) = \begin{vmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cot B & 1 \\ \sin^2 C & \cot C & 1 \end{vmatrix}$, then :

(A) $\tan A + \tan B + \tan C$

(B) $\cot A \cot B \cot C$

(C) $\sin^2 A + \sin^2 B + \sin^2 C$

(D) 0

***109.** The roots of the equation $\begin{vmatrix} {}^x C_r & {}^{n-1} C_r & {}^{n-1} C_{r-1} \\ {}^{x+1} C_r & {}^n C_r & {}^n C_{r-1} \\ {}^{x+2} C_r & {}^{n+1} C_r & {}^{n+1} C_{r-1} \end{vmatrix} = 0$ are :

(A) $x = n$

(B) $x = n + 1$

(C) $x = n - 1$

(D) $x = n - 2$

- *110.** If $f(x) = \begin{vmatrix} 3 & 3x & 3x^2 + 2a^2 \\ 3x & 3a^2 + 2a^2 & 3x^3 + 6a^2x \\ 3x^2 + 2a^2 & 3x^3 + 6a^2x & 3x^4 + 12a^2x^2 + 2a^4 \end{vmatrix}$, then :
- (A) $f'(x) = 0$ (B) $y = f(x)$ is a straight line parallel to x-axis
(C) $\int_0^2 f(x)dx = 32a^4$ (D) None of these
- *111.** If $\begin{vmatrix} yz - x^2 & zx - y^2 & xy - z^2 \\ xz - y^2 & xy - z^2 & yz - x^2 \\ xy - z^2 & yz - x^2 & zx - y^2 \end{vmatrix} = \begin{vmatrix} r^2 & u^2 & u^2 \\ u^2 & r^2 & u^2 \\ u^2 & u^2 & r^2 \end{vmatrix}$, then :
- (A) $r^2 = x + y + z$ (B) $r^2 = x^2 + y^2 + z^2$
(C) $u^2 = yz + zx + xy$ (D) $u^2 = xyz$
- *112.** Let $f(n) = \begin{vmatrix} n & n+1 & n+2 \\ {}^nP_n & {}^{n+1}P_{n+1} & {}^{n+2}P_{n+2} \\ {}^nC_n & {}^{n+1}C_{n+1} & {}^{n+2}C_{n+2} \end{vmatrix}$, where the symbols have their usual meanings. Then $f(n)$ is divisible by :
- (A) $n^2 + n + 1$ (B) $(n+1)!$ (C) $n!$ (D) None of these
- *113.** If a, b, c are non-zero real numbers such that $\begin{vmatrix} bc & ca & ab \\ ca & ab & bc \\ ab & bc & ca \end{vmatrix} = 0$, then :
- (A) $\frac{1}{a} + \frac{1}{b\omega} + \frac{1}{c\omega^2} = 0$ (B) $\frac{1}{a} + \frac{1}{b\omega^2} + \frac{1}{c\omega} = 0$
(C) $\frac{1}{a\omega} + \frac{1}{b\omega^2} + \frac{1}{c} = 0$ (D) None of these
- *114.** The values of $k \in R$ for which the system of equations $x + ky + 3z = 0$, $kx + 2y + 2z = 0$, $2x + 3y + 4z = 0$ admits of nontrivial solution is :
- (A) 2 (B) $5/2$ (C) 3 (D) $5/4$
- *115.** The determinant $\begin{vmatrix} \cos(\theta + \phi) & -\sin(\theta + \phi) & \cos 2\phi \\ \sin \theta & \cos \theta & \sin \phi \\ -\cos \theta & \sin \theta & \cos \phi \end{vmatrix}$ is :
- (A) Positive (B) Independent of q
(C) Independent of ϕ (D) None of these

***116.** The system of equations $-2x + y + z = a$, $x - 2y + z = b$, $x + y - 2z = c$ has :

- (A) no solution if $a + b + c \neq 0$
 (B) unique solution if $a + b + c = 0$
 (C) infinite number of solutions if $a + b + c = 0$
 (D) None of these

***117.** $\Delta = \begin{vmatrix} a & a^2 & 0 \\ 1 & 2a+b & (a+b) \\ 0 & 1 & 2a+3b \end{vmatrix}$ is divisible by :

- (A) $a+b$ (B) $a+2b$ (C) $2a+3b$ (D) a^2

118. Number of 2×2 matrices formed using elements from the set $\{0, 1, 2, 3, -1, -2, -3\}$ which are symmetric or skew symmetric is :

- (A) 350 (B) 349 (C) 348 (D) 343

119. Let $A = (a_{ij})_{3 \times 3}$, $B = (b_{ij})_{3 \times 3}$, $C = (c_{ij})_{3 \times 3}$ where b_{ij} = cofactor of a_{ij} in A, c_{ij} = cofactor of b_{ij} in B then $|C| =$

- (A) $|A|$ (B) $|A|^2$ (C) $|A|^3$ (D) $|A|^4$

120. If $a, b, c, d, e, f \in I$ (set of integers), then $\begin{vmatrix} 0 & a & b & c \\ -a & 0 & d & e \\ -b & -d & 0 & f \\ -c & -e & -f & 0 \end{vmatrix}$ is :

- (A) always prime (B) always composite
 (C) always even (D) always perfect square

***121.** Let α, β, γ be the roots of $x^3 + 2x^2 + 2x + 1 = 0$, $A_n = \alpha^n + \beta^n + \gamma^n, n \in N$, and $D = \begin{vmatrix} A_1 & A_2 & A_3 \\ A_2 & A_3 & A_4 \\ A_3 & A_4 & A_5 \end{vmatrix}$ then :

- (A) $\arg(D) = \pi$ (B) $D = -3$ (C) $D = 3$ (D) $\arg(D) = 0$ 

122. Let α_1, α_2 and β_1, β_2 be the roots of $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$, respectively. If the system of equations $\alpha_1 y + \alpha_2 z = 0$ and $\beta_1 y + \beta_2 z = 0$ has a nontrivial solution, then prove that $b^2 pr = q^2 ac$.

123. Evaluate $\begin{vmatrix} {}^x C_1 & {}^x C_2 & {}^x C_3 \\ {}^y C_1 & {}^y C_2 & {}^y C_3 \\ {}^z C_1 & {}^z C_2 & {}^z C_3 \end{vmatrix}$.

124. Prove that $\begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix} = 4(b+c)(c+a)(a+b)$

- 125.** Show that the solutions of the equation $\begin{bmatrix} x & y \\ z & t \end{bmatrix}^2 = O$ are $\begin{bmatrix} x & y \\ z & t \end{bmatrix} = \begin{bmatrix} \pm\sqrt{\alpha\beta} & -\beta \\ \alpha & \mp\sqrt{\alpha\beta} \end{bmatrix}$, where α, β are arbitrary.
- 126.** If $A = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}$, then prove that $A^2 + 3A + 2I = O$. Hence, find B and C matrices of order 2 with integer elements, if $A = B^3 + C^3$
- 127.** If B and C are nonsingular matrices and O is null matrix, then show that $\begin{bmatrix} A & B \\ C & O \end{bmatrix}^{-1} = \begin{bmatrix} O & C^{-1} \\ B^{-1} & -B^{-1}AC^{-1} \end{bmatrix}$.
- 128.** Show that every square matrix A can be uniquely expressed as $P + iQ$, where P and Q are hermitian matrices.
- 129.** Express A as the sum of a Hermitian and a skew-hermitian matrix, where $A = \begin{bmatrix} 2+3i & 2 & 5 \\ -3-i & 7 & 3-i \\ 3-2i & i & 2+i \end{bmatrix}$.
- 130.** The value of $|\alpha|$ for which the system of equation $\alpha x + y + z = \alpha - 1$, $x + \alpha y + z = \alpha - 1$, $x + y + \alpha z = \alpha - 1$ has no solution, is _____.
- 131.** The sum of values of p for which the equation $x + y + z = 1$, $x + 2y + 4z = p$, and $x + 4y + 10z = p^2$ have a solution is _____.
- 132.** If three distinct points $P(3u^2, 2u^3)$, $Q(3v^2, 2v^3)$, and $R(3w^2, 2w^3)$ are collinear, then $uv + vw + wu$ is equal to _____.
- 133.** Let $D_1 = \begin{vmatrix} a & b & a+b \\ c & d & c+d \\ a & b & a-b \end{vmatrix}$ and $D_2 = \begin{vmatrix} a & c & a+c \\ b & d & b+d \\ a & c & a+b+c \end{vmatrix}$ then the value of $\left| \frac{D_1}{D_2} \right|$, where $b \neq 0$ and $ad \neq bc$, is _____.
- 134.** If $\Delta = \begin{vmatrix} 1 & 3\cos\theta & 1 \\ \sin\theta & 1 & 3\cos\theta \\ 1 & \sin\theta & 1 \end{vmatrix}$, then the value of $(\Delta_{\max})/2$ is _____.
- 135.** If $\begin{vmatrix} x & x+y & x+y+z \\ 2x & 3x+2y & 4x+3y+2z \\ 3x & 6x+3y & 10x+6y+3z \end{vmatrix} = 64$, then the real value of x is _____.
- 136.** If $a_1, a_2, a_3, \dots, a_{12}$ are in A.P. and $\Delta_1 = \begin{vmatrix} a_1a_5 & a_1 & a_2 \\ a_2a_6 & a_2 & a_3 \\ a_3a_7 & a_3 & a_4 \end{vmatrix}$, $\Delta_3 = \begin{vmatrix} a_2a_{10} & a_2 & a_3 \\ a_3a_{11} & a_3 & a_4 \\ a_3a_{12} & a_4 & a_5 \end{vmatrix}$ then $\Delta_1 : \Delta_2 =$ _____.
- 137.** If $\begin{vmatrix} x^n & x^{n+2} & x^{n+4} \\ y^n & y^{n+2} & y^{n+4} \\ z^n & z^{n+2} & z^{n+4} \end{vmatrix} = \left(\frac{1}{y^2} - \frac{1}{x^2} \right) \left(\frac{1}{z^2} - \frac{1}{y^2} \right) \left(\frac{1}{x^2} - \frac{1}{z^2} \right)$ then $-n$ is _____.
- 138.** Given $A = \begin{vmatrix} a & b & ac \\ d & e & 2f \\ l & m & 2n \end{vmatrix}$, $B = \begin{vmatrix} f & 2d & e \\ 2n & 4l & 2m \\ c & 2a & b \end{vmatrix}$, then the value of B/A is _____.

139. If $(1+ax+bx^2)^4 = a_0 + a_1x + a_2x^2 + \dots + a_8x^8$, where $a, b, a_0, a_1, \dots, a_8 \in R$ such that $a_0 + a_1 + a_2 \neq 0$

and $\begin{vmatrix} a_0 & a_1 & a_2 \\ a_1 & a_2 & a_0 \\ a_2 & a_0 & a_1 \end{vmatrix} = 0$, then the value of $5\frac{a}{b}$ is _____.

140. If $A = \begin{bmatrix} 5 & -2 \\ 8 & -3 \end{bmatrix}$, and $A^{2016} = \lambda A - \mu I$ then $\lambda - \mu$ is _____.

141. Consider the system of linear equations

$$\sin(3\theta)x - y + z = 0$$

$$\cos(2\theta)x + 4y + 3z = 0$$

$2x + 7y + 7z = 0$ has non-trivial solutions. Number of values of $\theta \in (-2\pi, 2\pi]$ is _____.

142. Let A be 4×4 matrix such that $a_{ij} = \frac{1}{\min(i, j)}$ then sum of digits of absolute value of $\frac{1}{|A|}$ is _____.

143. Let $f(x) = (1-x+x^2)e^x$ then absolute value of $\begin{vmatrix} f(0) & f^I(0) & f^{II}(0) \\ f^I(0) & f^{II}(0) & f^{III}(0) \\ f^{II}(0) & f^{III}(0) & f^{IV}(0) \end{vmatrix}$ is _____.



$f^r(0)$ denotes r^{th} derivative of f at 0

144. Let $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & 4 \\ -1 & 0 & -1 \end{bmatrix}$ then number of factors of Trace of A^3 is _____.

145. Let A be an orthogonal matrix (i.e. $AA' = I$), of order 5 & $|A| = 1$ then $|A - I|$ is _____.

146. If $\alpha \neq a, \beta \neq b, \gamma \neq c$ & $\begin{vmatrix} \alpha & b & c \\ a & \beta & c \\ a & b & \gamma \end{vmatrix} = 0$ then $\frac{\alpha}{\alpha-a} + \frac{\beta}{\beta-b} + \frac{\gamma}{\gamma-c}$ is _____.

